Chapter 6 Combinatorics Review

Terms

Fundamental Counting Principle: if one item can be selected m ways and another item can be selected n ways, then the two items can be selected mn ways.

Permutations: an ordered arrangement of items. The number of permutation of n distinct items is $n!$ The number of permutations of n distinct objects taken r at a time is $_nP_r = \dfrac{n!}{(n-r)!}$. The number of permutations of n objects with a objects alike, b objects alike, c alike, …. is $\dfrac{n!}{a!b!c!...}$.

Combinations: an unordered arrangement of items. The number of combinations of n objects taken r at a time is $_nC_r = \dfrac{n!}{r!(n-r)!}$

Pascal's Triangle

Any term on any row: $_{r-1}C_{t-1}$

Symmetrical pattern: $_nC_r = {_nC_{n-r}}$

Recursive pattern: $_nC_r = {_{n-1}C_{r-1}} + {_{n-1}C_r}$

Binomial Theorem

Expansion using combinations: $(a+b)^n = {_nC_0}a^n + {_nC_1}a^{n-1}b^1 + {_nC_2}a^{n-2}b^2 + {_nC_3}a^{n-3}b^3 ... {_nC_n}b^n$

For the $(k+1)$ term: $t_{k+1} = {_nC_k}a^{n-k}b^k$

Review Questions

1. Determine the number of four digit numbers if:

    a) Repeats are allowed
    b) Repeats are not allowed
    c) No consecutive digit can be repeated
    d) The first two digits are odd numbers and the rest are greater than the number five

2. Determine the number of license plates possible with three letters followed by three digits if:

    a) Repeats are allowed
    b) Only consonants and even digits are allowed

3. The Commercetown Super Food Store has a bar code system for pricing their products. All products begin with the digits 4 5192. Produce items then have the digits 016 followed by seven digits. The example below is for bulk carrots.

4 5192 016 0002419

    How many different items of produce can the store sell?

4. Determine the number arrangement of the letters:

    a) CANISTER
    b) PARALELLOGRAM

5. Determine the number of four letter arrangements of the letters:

    a) VALUED
    b) COMBINED

6. Thelma and Bill are members of scout group with ten members. How many ways can the group line up if Thelma is in the front of the group and Bill is at the end?

7. How many ways can a president, vice-president and a secretary be selected from a class of thirty-two students?

8. The Terry Fox Girls Soccer team plays a nine game regular season. How many ways can the team end its season with five wins, three losses and a tie?

9.   Six boys and five girls stand in a line. How many arrangements are possible if all of the girls must stand together?

10.   A survey of grade 12 students is conducted to select music for grad night. From the survey shown below, how many different ways can the survey be answered?

Place a check mark beside any of the types of music listed below. You may check as few or as many of the types of music listed below.

___ Hip-hop
___ Rap
___ Rock
___ Disco
___ Punk
___ Heavy metal
___ Latin dance
___ Grunge

11.   Karen has a collection of her favourite novels on a bookshelf. The collection consists of three science-fiction, four romance, two action-adventure and five detective novels. How many ways can the collection be arrange on the shelf if the books must stay with their genre?

12.   Determine the number of paths that can be drawn from point A to point B without backtracking.

a)
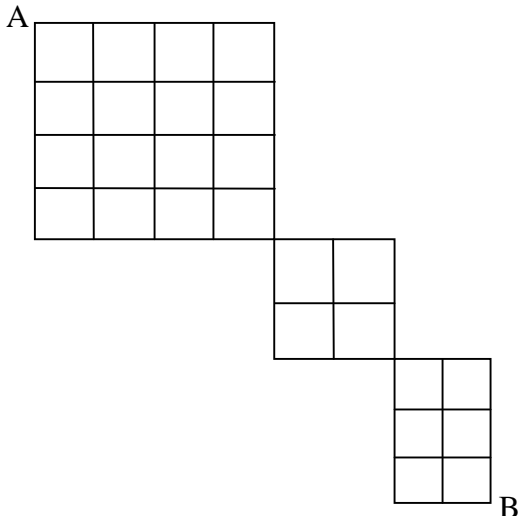
b)
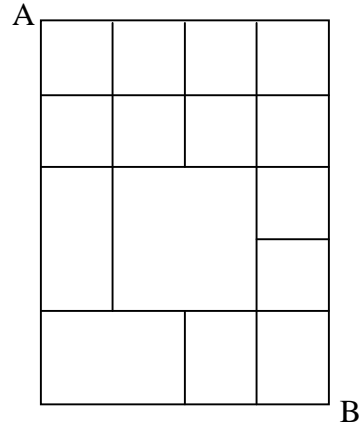
13. Determine the number of ways that a committee of three people can be selected from a group of twelve girls and eleven boys if:

    a)   The committee must have exactly two girls
    b)   The committee must have at most one girl
    c)   The committee must have at least one member from each sex
    d)   There are no restrictions on the makeup of the committee

14. How many ways can a president, vice-president and a committee of five people be selected from a group of twenty-four people?

15. A school committee consists of one school vice-principal, three student council member and two teachers. How many possible committees could be formed in a school with three vice-principal, eight student council members and eighty teachers?

16. To graduate from a high school, students must take one of three English 12 courses and at least one of a Mathematics 12, Chemistry 12, Physics 12 and Biology 12 courses. How many ways can a student graduate?

17. A fashion designer is putting together a display from a collection of his latest fall, winter, spring and summer clothing lines. He must choose two items each from each season and then arrange the clothing in the display. How many arrangements are possible if his fall clothing line has six items, his winter clothing line has thee items, his spring clothing line has eight items and his summer clothing line has five items?

18. Given the letters A B C D E F G H I, how many three digits codes are possible if the code must have at least one vowel?

19. Determine the number of five card hands possible:

    a)   With exactly two red face cards
    b)   With at most one face cards
    c)   At least one face card

20. Eight dots are spaced evenly about the circumference of a circle. Lines can be drawn by joining two points. How many different lines can be drawn?

21. How many different lines can be drawn by connecting vertices of an octagon?

22. Using $_nC_r$ notation:

    a)   Determine the seventh term on the twelfth row
    b)   Express the term from above as the sum of two $_nC_r$ notations
    c)   Express the term from above with an equivalent $_nC_r$ notation

23. Prove the recursive pattern of Pascal's Triangle algebraically.

24. Solve algebraically.

   a) $_nP_2 = 30$ b) $_nC_2 = 10$

   c) $\dfrac{(n+2)!}{n!} = 30$ d) $\dfrac{3n!}{(n-3)!} = 216n$

25. Determine the number of terms in the expansion $(2x - 3y)^6$.

26. Expand $(2a - 3b)^3$.

27. Determine the third term of the expansion $(x + 2y)^5$.

28. For the expansion of $(x + by)^5$, one of the terms is $-80x^2y^3$. Determine the value of b.

29. For the expansion of $(3x - 4y)^7$, determine the value of the coefficient for the term containing y^2.

30. Given $(x - 1)^7$, what is the sum of the coefficients of its expansion?

31. Evaluate $\displaystyle\sum_{n=1}^{4} {_5P_n}$.

Solutions

1. a) 9 000   b) 4 536   c) 6 561   d) 400

2. a) 17 576 000   b) 1 157 625

3. 10^7

4. a) 40 320   b) 86 486 400

5. a) 360   b) 840

6. 40 320   7. 29 760   8. 504

9. 604 800   10. 256   11. 829 440

12. a) 4 200   b) 58

13.   a)   726   b)   825   c)   1 386   d)   1 771

14.   14 536 368   15.   530 880   16.   45

17.   508 032 000   18.   384

19.   a)   227 700   b)   1 754 688   c)   1 940 952

20.   28   21.   20

22.   a) $_{11}C_6$ b) $_{10}C_5 + _{10}C_6$ c) $_{11}C_5$

23. $_nC_r = _{n-1}C_{r-1} + _{n-1}C_r$

$$\frac{n!}{r!(n-r)!} = \frac{(n-1)!}{(r-1)!(n-1-(r-1))!} + \frac{(n-1)!}{r!(n-1-r)!}$$

$$\frac{n!}{r!(n-r)!} = \frac{(n-1)!}{(r-1)!(n-r)!} + \frac{(n-1)!}{r!(n-r-1)!}$$

$$\frac{n!}{r!(n-r)!} = \frac{r(n-1)!}{r!(n-r)!} + \frac{(n-r)(n-1)!}{r!(n-r)!}$$

$$\frac{n!}{r!(n-r)!} = \frac{r(n-1)! + (n-r)(n-1)!}{r!(n-r)!}$$

$$\frac{n!}{r!(n-r)!} = \frac{(n-1)!(r+(n-r))}{r!(n-r)!}$$

$$\frac{n!}{r!(n-r)!} = \frac{(n-1)!(n)}{r!(n-r)!}$$

$$\frac{n!}{r!(n-r)!} = \frac{n!}{r!(n-r)!}$$

24.   a)   6   b)   5   c)   4   d)   10

25.   7   26. $8a^3 - 36a^2b + 54ab^2 - 27b^3$

27. $40x^3y^2$ 28. -2 29.   81 648

30.   0   31.   205